The Dream Girl

by Doug Muder

The problem, stated in its simplest form, was how to keep body and soul together through the summer.

I had just finished my sixth year in a doctoral program which, according to the Department's brochures, was supposed to be completed after four, and everyone (myself most of all) was becoming impatient with me. During the regular academic year the Department's impatience had no financial implications, because the campus was overrun by mindless hordes of undergraduates, many of whom had been told (probably incorrectly) that they needed to learn calculus. Forced to offer more courses than the professors felt obligated to teach, the Department needed grad students like me. Truth to tell, it actually preferred the more experienced grad students, because we had done it all before and weren't likely to cause the kind of problems that show up whenever someone teaches for the first time. Occasionally my Nth-year colleagues and I would get together over beer, and speculate that the Department was slowing down our progress intentionally so that it could continue to exploit our cheap labor.

But summer was another matter. A few undergrads hung around for the summer term, and a few courses had to be run to keep them busy. But the supply-and-demand of the situation was reversed: more grad students needed support than there were courses to teach. Those coveted summer teaching slots naturally went to the students who were still "on track," as the Department put it.

I was off track. I had finished the last of my exams and course work two years before, but my thesis research refused to come together. Not that I was stuck – that's an important distinction. I have known several very bright people who did get stuck on their thesis research. They all fit one of two patterns: either they got depressed and did nothing, or they filled their days with hundreds of good and bad habits (including teaching) that prevented them from getting down to business. When they did work on their thesis problems, they went through the same thought patterns again and again, like rats who somehow keep imagining that the maze will be different this time.

My experience was diametrically opposite. I was fascinated by my thesis problem, and every day or two I discovered some interesting new fact about it, though (fascinating as they were) these insights and discoveries stubbornly refused to add up to a solution. As I got further and further off track, the problem took up more and more of my days, until even my busiest, most understanding friends began to complain that they never saw me any more. In the winter I had believed I was close to a breakthrough that would win me my honorable discharge by the end of the school year, so I had redoubled my efforts. By spring, as the inevitability of yet another year as a student became more and more apparent, I could barely spare time to eat and sleep and teach my class.

Whenever I reach this point in my story, my non-mathematical acquaintances frequently ask me what on Earth such a problem could be. Mathematics, according to the common belief, was all worked out by men with wigs and starched collars in the eighteenth century at the latest. When I say that I am a geometer people are even more puzzled, since Euclid surely knew all there was to know about geometry three centuries before Christ. A few of the more technically minded recall non-Euclidean geometry, which at least gets us into the nineteenth century.

I am, I'm afraid, not terribly good at popularization, which is to say, at concocting explanations that give non-specialists the mistaken impression that they understand something. This is the best I can do: (Bear in mind that you can skip this explanation entirely and be none the worse off for understanding my story.) There is a class of surfaces that are defined by a class of equations, both of which have properties that interest people who study prime numbers. There is a way of doing arithmetic with the points of these surfaces, of adding two to get a third (or, more likely, some virtual object that is not really a point at all, but never mind – this is a popularization, you don't need to know that). My advisor strongly believed, but could not prove – proof being the coin of the realm in mathematics – that for each such surface there is a particular, easily-defined set of points such that all the other points can be expressed as sums of these special points. This conjecture was part of a much larger structure of conjectures, which (if they would all turn out to be true and if he could find proofs of them) would give him the place in the history of mathematics that he thinks he deserves. My mission, which (if I could do it before our rivals in France) would get me my doctorate and give me the opportunity to have a successful academic career, was to prove this conjecture.

Usually, people who make it this far then ask: "Why would anyone want to know that?" The right answer to that question, or even a decent popularization of it, requires explaining the previous two hundred years of mathematical history. I have yet to meet a non-mathematician who can stay awake for it. All I can do is remind you that academics are a strange breed, and that there are men who have happily spent their lives trying to decide whether or not Francis Bacon wrote the plays of Shakespeare. Trust me, men have vanished into problems like mine, leaving behind nothing more than a trail of chalk dust and waste baskets full of calculations.

Fortunately, given some perverse view of fortune, I did not have a girl friend. Despite being somewhat geeky and socially awkward, I have had girl friends from time to time, most recently during my fourth year. Cynthia was a year ahead of me, and finished after her fifth year. She did well in the job market, landing a two-year post-doc at UCLA. At that time we were both imagining that I would also finish after my fifth year, and I promised to focus my job search on southern California so that we could be together. Needless to say, that didn't happen. We never exactly broke up, but the frequency of our phone calls diminished exponentially with time. Our vague plans to spend Christmas break together never converged on a specific destination. At some point I just stopped thinking of myself as a person with a girl friend. I'm sure she has found someone else by now.

That was (as I was saying) fortunate, because none of my girl friends – even the ones who were also in mathematics – had ever tolerated my sinking into a math problem to even a fraction of the degree that I had sunk into this one. Women, I have discovered, need attention the way that plants need sunlight. But unlike plants, they do not just wither away when they don't get it. Instead, they become increasingly angry and demanding. I had no attention to give, and so I was grateful not to have any one mad at me.

But I had to eat. And, while my room wasn't much, the landlord was not inclined to let me keep it for nothing. I had heard stories about grad students who went homeless for a summer, sleeping in remote corners of the library stacks and showering at the gym, but I wasn't inclined to try it.

So I needed a job.

The ideal real-world job for a mathematician wrestling with a research problem is

lighthouse keeper – something remote that leaves a lot of space for thinking. There are, however, very few lighthouses in the Midwest, and I suspected that the few that might exist on the Great Lakes had already been automated. When I was an undergrad it had been common for math students to find summer jobs programming computers, but more recently programming had turned into a real profession that required credentials and experience, neither of which I had. Because I hadn't been reading newspapers for the past six months, I was surprised to discover that the economy was in recession and even a burger-flipping job would be hard to come by.

By mid-June, I was starting to wonder just how bad a sleeping bag in the stacks could be. (The carpeting provided a little padding. The library's bathrooms were clean. All over the world there are refugees who would kill for such luxury.) I also had begun to notice the pan-handlers that I passed on the street, and wonder whether they were actually destitute or just extremely talented at appearing to be destitute. Could an amateur compete in that market, or was there more to it than met the eye?

And then, just before exiting to spend the summer with colleagues at Harvard, my advisor suggested something. He had a friend who had a friend who would soon be leaving for Europe.

House sitting, I thought. *Please, Lord, let someone pay me to live in his house. Let it have a moat and a drawbridge and a cellar so deep you can't even hear the cars go by.*

This friend of a friend had once been chairman of the chemistry department, and had only recently retired. He felt a substantial obligation to his mentor, a man who was now in his eighties and quite infirm, but who was legendary in his field. (I, of course, understood what this meant. Academics is perhaps the only area of human endeavor in which a man can be legendary without his name being recognized by anyone whose office is more than two floors away. I had no doubt that the name of this legendary chemist would mean nothing to me, and I was correct.) He was too infirm, it seemed, to accompany the friend-of-a-friend to Europe.

I began having visions of a summer spent changing adult diapers. Despite the fact that I had skipped breakfast and was overdue for lunch, these visions ruined my appetite to the point where the alternative of a season-long fast began to seem reasonable.

But no, the legendary chemist's physical needs were being well tended in a nursing home not far from campus. His problem was a degenerative mental condition that caused him to sleep a great deal of the time, and to lapse into fantasies when he was awake. Doctors had told the friend-of-a-friend (let's call him F) that the legendary chemist (C) needed regular mental stimulation. For that reason, F had been visiting C daily for the past two years and trying to engage his mind. F had no doubt that the nursing staff was dedicated and excellent in its way, but he was afraid that he would come back from Europe in the fall only to find that C had lapsed into a dream world from which he would never return.

And so, F was willing to pay someone – me, presumably – to visit C for a few hours a day until F came back. A needy high school student wouldn't do. The visitor had to be able to engage C's mind on something near to its own level, and tempt it to remain focused on reality. My advisor (A) quoted a figure which would require a little belt-tightening, but was quite generous for part-time work that involved no heavy lifting or diaper-changing.

I asked, somewhat foolishly perhaps, if F had considered asking a chemistry student, who might recognize C's name and feel honored to spend time with him for free. I was

surprised to discover that A had made the same suggestion, but that F had rejected it. C had specifically requested a mathematician. Mathematicians, C had said, were "more imaginative" than chemists. F already felt guilty enough about deserting his mentor for the summer, so he was inclined to humor him in this. "It makes a certain amount of sense," A commented to me. "After all, it's not like he has a lab out there. He just has his mind."

"Part of it, anyway," I joked.

The job required an audition, though F (A assured me) was nearly as desperate to get to Europe as I was to find a job. So unless I caused the old man bodily harm I was probably in.

I used A's phone to call F and arranged to meet him at the nursing home later that afternoon. F had one of those marvelously neutral accents that Americans never manage to achieve, so I assumed that he was a European of some sort, and that the summer trip was a return to the old country. He gave me the address of the nursing home. It was just walking distance from campus, but in a direction I had never walked. That came as something of a surprise to me, because I have the habit of walking when I am trying to digest a difficult idea. After six years, I believed that I had wandered everywhere that a person could wander from campus. But clearly I had not.

Meeting strangers always makes me nervous, particularly when I want something from them. Being nervous makes me worry about time, and so I left the library more than half an hour earlier than should have been necessary. But it was necessary, because my first two attempts to cut through the park both failed, causing me to come out first southeast and then northeast of my intended destination. I was already five minutes late when I came out the third time and gratefully spotted the nursing home right in front of me.

The home was one of those Gilded Age mansions that no one could afford to live in any more. Or perhaps those who could afford it no longer wanted to live in the city. It was made of yellowish brick and had the irregular sort of architecture that seems to have been popular among the robber barons. Extra wings jutted out at odd angles and rounded, Rapunzel-like towers erupted apparently at random. A semi-circular driveway intersected the street in two places. The exact apex of the drive was covered by a canopy that led to the main entrance.

A tall, thin, gray-haired man in a well-tailored suit was standing under the canopy, not far from a Mercedes. He looked like a very busy person who (by dint of good breeding and excellent education) was able to affect a pose of nonchalance. "You must be Mr. Murphy," he said as I approached.

I apologized for my lateness and somewhat clumsily shook the hand he had offered. I still could not guess his country of origin, though I was leaning towards eastern Europe rather than western. Perhaps F had been a Cold War refugee or defector.

He led me into the building, past a security guard who seemed more interested in the possibility that some unauthorized person might leave rather than enter. "I'm sure this meeting is merely a formality," F said to me. "But I wanted Andrew to know that I am not forcing you on him. He's really quite a remarkable man. I have no doubt you'll hit it off marvelously."

The nursing home's front room resembled the lobby of a small hotel, with a nurse's station replacing the front desk. Next to the nurse's station was a set of automatic double doors, presumably leading to the residents' rooms. To one side of the main entrance, a collection of empty chairs surrounded a television that was turned up far too loud. An ancient woman with a walker stood at an angle that allowed her to watch either the television or the entrance with

equal ease.

F indicated that I should stay in the lobby, and that he would go collect C by himself. He vanished through the automatic double doors. I stood there, soaking up the air conditioning and hoping that the sweat I had worked up walking over would evaporate before I met C. A clattering behind me caused me to turn my head: it was the old woman. She made eye contact for a moment and then broke it off to continue on her way through the doors.

"He's a loon, that one," she said, her head facing away from me. "Don't let him get on about his daughter. He hasn't *got* a daughter. None of us do. Do you think we'd be here if we had anybody?" She laughed bitterly, then coughed a few times, and then the doors closed behind her.

I stared at the ceiling for a while, thinking about the arithmetic of points on surfaces, until the doors opened again and F appeared pushing a wheel chair. The man in the chair was dressed in pajamas and a robe, but he was cleanly shaved and looked quite alert.

"Mr. Murphy," F said, "this is Andrew Simon."

He said the name as if a flourish of trumpets were sounding somewhere, so I responded in kind. "It's an honor," I said. And I supposed it probably would have been an honor, if I hadn't given up on chemistry after my freshman year.

Andrew laughed at me in a gentle sort of way. "Senile old men don't honor anyone," he said. "At least not intentionally."

F waved that comment away, and then gestured for me to take the handles of the chair. "I think the best way for you to get acquainted, Andrew, is to show Mr. Murphy your afternoon walk." And then he said to me, "We do it every day if the weather allows. If it doesn't, I wheel him through the halls. It's good for him to get a change of scene. I'll wait here for you."

Andrew gestured his acceptance of this plan, so I began pushing the chair towards the main entrance. The security guard glanced over at F as we approached, and then said "Good afternoon, Mr. Simon."

When we were safely outside, Andrew said, "The poor man's scared to death."

"The guard?"

"Max." He meant F. "He used to believe that insanity wasn't contagious, but now he's not so sure. He's going to run away to Europe before it's too late."

Not far from the canopy was a sidewalk that led through an archway into a courtyard. There was a fountain in the courtyard, and a number of wheelchairs were parked around it.

"This is much nicer than the home my grandmother was in," I said, trying to make conversation. I wasn't sure this was such a good opening. If he followed up by asking how my grandmother was doing, I would have to admit she was dead, which could only lead to an awkward silence.

"They tell me I can afford it," he said. "That means they expect me to die before the money runs out."

And so we arrived at the awkward silence by an alternate route. Beyond the courtyard was another archway, and a gravel path leading into the woods. I didn't understand how there could possibly be woods here. The park was on the other side of Temple Avenue, and I was sure there was nothing on this side but the grid of city streets. I resolved to look at a map the next time I was in the library.

"Max tells me you're a mathematician, a graduate student."

I acknowledged that Max was correct.

"What are you working on?"

I gave him my usual explanation, about surfaces and points and arithmetic. I was hoping that he wouldn't ask me why anyone would care, because I was sure I couldn't connect it to anything in chemistry. He didn't.

"How do you go about trying to prove something like that?"

"Well," I answered cautiously, unsure of just how much detail he could follow or would want to hear, "the most obvious way is to do a proof by contradiction. If the conjecture is wrong, then there is some surface in the class and some point on that surface that *can't* be expressed as a sum of the standard points. That would have to be a very special surface and a very special point. So you can start making deductions about just how special they have to be. If you do it right, you should get a set of conditions so restrictive that they contradict each other."

"So to prove that such a surface doesn't exist, you imagine that it does and start visualizing what that would mean."

"Right," I responded.

"And you keep fleshing that vision out, getting a clearer and clearer picture of the world that surface would have to live in, until finally you can see what's wrong with that world, why it can't exist."

I was surprised that he was able to express the idea so concisely. "That's right."

He laughed. "So you spend all your time thinking about things that don't exist."

I acknowledged that he was correct.

"That's quite delusional, you know," he said. His amusement was contagious; I started smiling myself. "If you don't watch out, you'll find yourself in a place like this one."

"I'm afraid I couldn't afford it," I replied.

"Then you'd best stay sane."

There wasn't anyone else on the path. In the shade of the trees, the June day was very pleasant. I thought to myself that I might enjoy having an excuse to get out in the daylight a few hours each afternoon. Before long we came to a pond, which made me renew my vow to study a map. Next to the pond was a bench. Andrew had me stop his chair beside the bench and sit down. Out in the water some ducks started heading in our direction.

"Max and I never feed them," he said. "But they never learn."

We sat silently for several minutes. The ducks serpentined closer to us, but didn't come out of the water.

"What if they do exist?" he asked.

"What?" I had gotten lost in my mathematics again.

"The surface and the point. What if they do exist?"

"Then my advisor's conjecture is wrong."

"Conjectures often are," he said. "Don't you think you'd have found the contradiction by now if there really were one?"

In my own mind, I had to confess that I'd been thinking the same thing since some time in mid-winter. Every few days I proved some new fact about that non-existent point and surface, but the facts I proved kept refusing to contradict each other.

"That's what happened to the non-Euclideans, wasn't it?" he asked.

It was. Back in the eighteenth century many people had tried to prove Euclid's parallel postulate, which Euclid had only assumed. The postulate said that if you had a line and a point not on the line, then there was exactly one other line that contained the point and was parallel to the first line. They had tried assuming that the postulate was false, and showing that if either two lines or no lines through the point were parallel to the first line, then some contradiction would have to follow. Either assumption seemed ridiculous, and a host of equally ridiculous-looking consequences followed from either one. But as nonsensical as these consequences appeared, they steadfastly refused to contradict any known theorem that didn't already assume the parallel postulate.

Eventually, early in the nineteenth century, someone -- I believe it was Gauss, but two or three other people have a more-or-less equivalent claim to the discovery -- started to believe that there was no contradiction to be found. The apparently nonsensical things they were proving were simply the true theorems of a *different geometry*, one that ran counter to our human intuitions, but was as logical and precise as anything Euclid had discovered.

"They thought," Andrew continued, "that they were thinking about things that didn't exist, and that if they thought about them long enough they would start seeing *why* they didn't exist. But eventually they fell down the rabbit hole and started believing in them."

"And they were right," I commented.

"Yes," he said thoughtfully. "They were."

The ducks were starting to lose interest in us.

"I joke about going down the rabbit hole," he continued, "But did you ever wonder if non-Euclidean geometry was part of what inspired Lewis Carroll in the first place? You think about nonsense long enough, and then – pop – there you are, on the other side of the looking glass, where it all looks sensible."

I appreciated that Andrew hadn't insulted my intelligence by reminding me that Carroll was a mathematician. His real name was Charles Dodgson, and he wrote his mathematical works under that name. I had heard an amusing story about his two identities, the mathematician and the story-teller. (Every time I tell the story, I promise myself that I will check that it's true before I tell it again. But I never do.) Apparently Queen Victoria had been quite taken with *Alice*, and had let it be known through certain discreet channels that it would please Her Majesty if the author would dedicate his next work to her. Any good monarchist would have been honored to receive such a request, but Dodgson seems to have had an ambivalent attitude towards the monarchy, or perhaps towards the entire Victorian Age. So he followed the letter of the request rather than the spirit. His next work, an impenetrable tome called *The Theory of Determinants*, was dedicated to the Queen.

We fell silent again. The sunlight reflected off the water in glistening patterns too complex to be described by anything short of chaos theory. I stopped thinking about Lewis Carroll and non-Euclidean geometry, and even about my thesis problem, which until that moment had been bubbling away on the back burners of my mind continuously since some time in the Cynthia era. Eventually I felt Andrew's hand on my arm, and heard him say, "Come on, I have someone I want you to meet."

The path continued in a direction I hadn't looked before. We walked up a hill and then along a rocky spine until we came to a spring. The spring created a tiny stream, which we followed until we came to a large open pasture. A lone tree stood in the middle of the pasture.

Under the tree, a young woman sat reading.

"Miranda!" Andrew called. "I've brought someone to meet you."

The woman put down her book and jogged in our direction. She was dressed like most of my students, in jeans and a t-shirt. Her hair was blacker than any I had ever seen, and as she approached I could see that her eyes were a startling violet.

"This is Mr. Murphy," Andrew announced.

"Daniel," I said, and wondered immediately if I should have said something less formal, like Dan or even Danny.

She stopped just far enough away that I didn't have to decide whether to shake hands or offer some other physical greeting.

"Mr. Murphy is going to be replacing Max," Andrew continued. "I'm afraid we're not going to be seeing him any more."

Miranda shrugged as if to say that it was no great loss to her. "This one is so much younger," she said. "I think I'm going to like him."

"I think I am too," Andrew said. "I'm hoping that he'll like us just as well."

For four years I had been teaching freshmen, and so I had trained myself not to be dazzled by the beauty of 19-year-olds. The stories of girls who try to trade sex for grades are (as far as I know) wildly exaggerated. But the dazzling ones know very well what effect they have on a young instructor, and are not above flashing a suggestive smile or glance to get an extra point or two of partial credit or avoid a penalty on a late assignment. Once one of the more daring ones looked over my shoulder while I graded her late homework, leaning so as to press her breasts up against my shoulderblade. A teacher who can't ignore this sort of thing belongs in another business, so I learned to ignore it.

But this meadow seemed to be a world away from the classrooms and offices where my defenses worked so well. And Miranda was indeed dazzling – more so, perhaps, than any calculus student I had ever taught. When she predicted that she would like me, I suddenly felt so happy it was frightening. I took a step backward, and then looked at the two of them standing together. I noticed that Andrew was wearing khaki pants now and a blue work shirt. "How," I said, "how are you standing here? Where is your chair? How did we get over that ridge?"

"Oh, that's easy," he said. "We're dreaming this."

My head snapped upwards and I took three quick, deep breaths. Sunlight was still strobing off the pond. Andrew was sitting peacefully in his chair.

"I'm sorry," I said, embarrassed. "I didn't mean to fall asleep. I was enjoying our conversation very much. I hope you won't …" and I trailed off, because I wasn't sure what I hoped. I hoped he wouldn't be offended. I hoped he wouldn't tell Max. I hoped he wouldn't torpedo my chance to get this job.

Andrew smiled at me, as if there was nothing for him to forgive.

"It's just," I said, "it's just that I've been working so hard lately."

"Too hard, I think. Sometimes you need to let up a little, get some distance from a problem."

"You're probably right," I said.

"The young researchers always try too hard. It's the disadvantage of youth. If you were older, your body just wouldn't stand for it. But you'll learn to relax. The world can't re-arrange

itself properly while you're staring at it."

I nodded, though I was still groggy and had no idea what he was talking about.

"Come along," he said. "Max probably isn't even worried about us yet."

The pond was actually closer to the house than I had remembered. A different collection of wheelchairs was arranged around the fountain in a different pattern. Max stood under the canopy, looking nonchalant.

"You've done quite well, Max," Andrew said as we approached. "I like young Daniel very much. I'm sure we'll get along famously."

Max exhaled in relief, which was the only time that I had detected even the smallest amount of tension in him. He told me that I was to get Andrew out of his room every afternoon for at least three hours, seven days a week, until Labor Day. He had left checks with the head nurse, who would give me one each Friday. We shook hands again, and then he took Andrew back inside.

Through June and into July, Andrew became the one regular part of my daily rhythm. With no class to prepare for or teach, I ate erratically, slept erratically, and talked to no one. At night I found unused classrooms in the math building and filled their blackboards with calculations. I bought enormous sheets of paper and covered them with diagrams as precise and detailed as the elaborate artworks of schizophrenics. I suspected that I was coming to know as much about my point and my surface as any human being had ever known about purely mathematical constructions.

I just didn't know whether they existed or not.

Andrew steadfastly believed that they did. "Anything you can see in that kind of detail has to exist," he said. "The mind itself calls it into existence."

That was a little too new-age for me, but all the same I appreciated his interest. It was good for me, talking to Andrew. He refused to get drawn into a line-by-line analysis of my calculations, and forced me to pull my conclusions up one or two levels of abstraction so that we could discuss them. He also got me out into the sunlight, and occasionally let me sleep for a half hour or so by the pond. It was usually the best sleep I got all day, though I didn't remember any more dreams.

Not that I was having trouble sleeping. In fact, I was sleeping more than I usually did, when you added it all up. Three hours here, four there. Sometimes I would wake up with my head on one of my giant diagrams and not know how long I had been asleep. Had I dreamed the last half hour of calculations or actually done them? Or had my hand done one set of calculations while my head dreamed a different set entirely?

If Andrew believed that my surface existed, my advisor believed just as adamantly that it didn't. After he left for Harvard I didn't see him, but I didn't need to. By now I had completely internalized his point of view. Alone in my room, I could argue with him to my heart's content, and point out every reason why he ought to believe in the existence of my surface. Eventually, I was able to argue just as vociferously with an internalized Andrew, and tell him all the reasons why the surface had to be nothing more than a well-worked-out figment of my imagination.

One day in late July, a thunderstorm kept us away from the pond. I tried wheeling Andrew through the halls, but we kept passing other men and women who had been set out in

the hall like inconvenient pieces of furniture while the staff cleaned their rooms. Their faces looked vacant, as if they saw neither the world in front of their eyeballs nor a more interesting one behind. We were both getting depressed. Finally I got an extra blanket and wrapped it around Andrew. We went outside, under the canopy, and watched the lightning and the big rain drops that bounced when they hit the pavement.

Eventually even that got old. "Take me back to my room," he said.

"But Max said – "

"Max isn't here."

It was a fact. There was no arguing with it. I wheeled him back into the building, through the automatic double doors and down the corridor to the elevator. Andrew's room was on the third floor. It was small, but very nice as such things go. He had a window overlooking the fountain and could see over the archway into the trees (whose existence I still had not verified with a map).

I had spent almost no time in Andrew's room. I gathered that Max had often come in to wake Andrew up and help him get ready for his excursion. But I rarely had to do anything more than just knock. Andrew would open the door immediately, as if he had been waiting. Sometimes he would meet me in the lobby or even under the canopy.

Andrew's window was dotted with raindrops. The natural light was gray and depressing, but the electric light made everything look yellow, which was worse. I turned it off again and wheeled him as close to the window as I could, then fiddled with the blinds to maximize the amount of light that came in.

There were no other chairs in the room, so I sat on the bed. "What do you do when you're here?" I asked.

"Sleep," he said. "Dream."

In the graying light, sleeping and dreaming seemed like desirable activities. But I remembered that my job was to keep Andrew away from his dreams, so I looked around for something to talk about. I noticed a picture on the table by his hand. "Is that your wife?" I asked. The woman in the picture was blond and middle-aged. She looked attractive, but in a serious way, like the author's portrait on the dust jacket of a book.

"Shelly," he said. "We were married for forty years. She died ten – no, eleven – years ago."

I hadn't known that. It made me realize just how little I knew about Andrew's life or he knew about mine. Talking about life just wasn't what we did together. I didn't know what Shelly had died of, or whether her death was the event that triggered his move to the nursing home. I looked to my right, at the picture on the end table next to the bed, and I stopped in mid-breath.

The picture was of a striking young woman with raven-black hair and violet eyes.

"Is this …," I fumbled for words, "… you and Shelly's daughter?"

"No," he said. "Shelly and I never had any children." He raised a hand. "No, don't try to sympathize. We never tried. We didn't want children because there was no one to raise them. I certainly wasn't going to do it. I had too much chemistry to do, too many thoughts in my head that needed to get out. And Shelly … Shelly was a lot like me. That's why we got along so well. She was a sociologist. She had papers to write and classes to teach, just like me. The woman in the picture, she was, well, it's a little hard to describe."

"A sister? A student?"

"No," he said. "Her name was Ariel. Pretty name, isn't it? It's one of the angelic names, you can tell by the 'el' at the end. I forget what it means."

I deduced (a beat late) that by "hard to describe" he meant that Ariel had been his mistress. She was certainly attractive enough, if the picture was anything close to the way she really looked. It wasn't my place to pry into Andrew's life, so I looked for some way to change the subject. I glanced around the room for some other object I could comment on. There weren't any. The room was strikingly barren: just a bed, a couple of tables, and pictures of two women.

"There's no way to explain what her picture is doing here without telling a very long story."

I tried to interpret his statement, to read between the lines. Was he telling me to move on, or was he asking me to inquire further? Anyone else probably would have known right away, but I had gotten so far out of the habit of talking to people that I had to think about it.

"I've got the time," I said tentatively.

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Ariel was probably about your age when I met her. It was twenty, maybe twenty-two years ago. She was much younger than me. My hair was already gray, and she was – well, just look at her. She was beautiful. You won't meet five women in your life who look that good, and four of them will be empty. All surface with nothing inside.

Ariel was full of life. Magical. No one uses the word *fey* any more, do they? They should; it means something.

We met in church, of all places. Neither of us belonged there. I had been an atheist since the age of about ten. Religion made people do strange things, I figured. They scared me, religious people, when you came right down to it. Still do. It's hard enough to know what the people around you are likely to do when they're just trying to make sense out of the world, but if they're also trying to guess what some ghost in the sky wants … well, you might as well give up. They could do anything.

Shelly, though, she felt differently. This was maybe the one place where we were mismatched. She had been raised in a church and it just felt right to her. She needed someplace to go on a Sunday morning, someplace where good people got together and talked about good things. She liked to sing the old hymns, and the poetry of the psalms really spoke to her, even the ones that sounded paranoid to me. "Enemies surround me, Lord, and they strike at me for no reason" – that sort of thing. I always thought, "King David, maybe you should spend less time complaining to God and more time trying to understand your enemies' motives." But not Shelly, she loved it.

So I humored her. She knew I was humoring her, and that was fine. We found a church that made at least half an attempt to be reasonable, one that didn't demand that I turn my back on the science that I did the other six-and-a-half days of the week, and we went. Every Sunday. I was the most faithful atheist you'll ever meet.

Eventually, after I had been a church-goer for quite a few years, Ariel showed up. She came every Sunday too. She didn't belong there either, but for the opposite reason. She had too many gods in her life. Like I said, she was fey. During the seventies she had been an honest-to-goddess witch. Dance naked in a circle, draw down the Moon – that kind of thing. She read

Tarot cards. She knew the locations of all the planets in her horoscope. She believed she could change the weather if she tried hard enough. I never argued with her about it. There didn't seem to be any point.

Years before, she had met George God-knows-where and married him God-knows-why. I tell you, if you ever figure out how women choose their mates, you'll have gotten far beyond me. He was an actuary, and was the most stick-in-the-mud man I have ever known. Good man, in his way. Honest. Hard working. Do anything for you if you were his friend. But Ariel's husband? I didn't pretend to understand it.

They had kids, two of them, a boy and a girl. They were beautiful, just like her. And they had the Devil in them, as my mother used to say about anyone who had a talent for enjoying life. Both of them knew how cute they were, and they knew that it was impossible to say no to them. I didn't even try. When we met Ariel and George, that was the end of me doing any adult socializing during the coffee hour after church. One or the other of the kids would drag me off somewhere and want me to do something. I taught the boy how to juggle, and he became pretty good after a while. Better than me. And the girl always wanted me to tell her stories. I didn't know that I knew any stories, but it turned out I did.

Our households started to grow together. We went out after church, spent holidays together, things like that. I think the kids needed grandparents. And, even though I never regretted deciding not to have children of our own, I loved having somebody else's. Shelly did too, I think. Eventually we had our own unofficial pew, on the left side of the center aisle, about halfway back. George sat on the aisle, then the kids, then Ariel, then me, and then Shelly on the inside, by the divider.

Couple-to-couple relationships always have one link that is stronger than the others, and with us it was the one between Ariel and me. There was nothing illicit about it – she and I just always had something to talk about. She flirted with me constantly, of course, but that was her way. Neither George nor Shelly ever objected to it. They thought we were cute together.

After we'd known them a couple of years, Ariel decided that she wanted to have a third child. But George was firm: two was enough. I think he didn't want to be outnumbered. She badgered, she wheedled, she bribed, but there was no moving him. "I'm not done," she kept saying. "There's still someone who needs to be here." But he wasn't giving in.

Every Sunday, after the first hymn, the kids would go downstairs to their classes, but Ariel didn't move into the empty space, closer to George. We stayed there in the center, hands at our sides, with just the edge of my hand touching the edge of hers. You can't imagine what that was like. You grew up in a free-wheeling age, where people jump in and out of bed all the time. But I didn't. My parents lived together until they died, and when I took a vow with Shelly I kept it. But every Sunday my little finger would rest next to Ariel's on the pew. It was special. It meant something.

And then one Sunday it meant something else.

It was almost spring, at the end of an ugly winter. The equinox was a week away, and that meant that it was Lent. Christian churches are at their most mysterious and ominous during Lent. They focus on suffering and sacrifice, two things that any sane person tries to avoid, in my opinion. The sermon was unusually boring that day, so I tuned it out. I was thinking about getting out a pen and doing some work on the back of the program, when Ariel leaned over and whispered, "Close your eyes."

I did. Within seconds I felt a hand on each side of my head, and I was being kissed. It was a wonderful, luscious kiss. I hadn't realized that I had any expectations about Ariel's kiss, but I did, and they were fulfilled.

For about half a second.

I was shocked, of course. We were in church. George was to her right, Shelly to my left. We couldn't be kissing.

We weren't.

When my eyes snapped open, I wasn't looking directly into Ariel's eyes, as I had expected, but at the altar in the front of the church. She was sitting where she always sat, to my right. The little finger of her left hand was gently, barely touching the little finger of my right. She smiled at me.

After the service I made up an excuse to go home immediately, without drinking coffee or playing with the kids. In the car, I felt as if I ought to confess something to Shelly, but I didn't know what. That I had a fantasy? She surely knew by now that I had fantasies. I certainly believed that she did, and I didn't grudge them to her. Physical monogamy is already more than enough to ask of people, without trying to control their thoughts. We both knew that.

So what had I done wrong?

I didn't say anything to Shelly, and I didn't talk to Ariel during the next week. I thought about playing sick on Sunday. On Wednesday I was convinced that I would, but by Saturday night it all seemed distant and silly. On Sunday morning the regular patterns took over, and I found myself sitting next to Ariel, my finger touching hers. We sang a hymn. The children left. We listened to announcements, sang another hymn, heard a reading. It all seemed interminable.

When the sermon started, I closed my eyes.

I felt her hand grip mine and lift, a signal to get up. I stood and noticed that the church was empty. We were alone – without ushers, ministers, parishioners, children, or spouses. She led me by the hand, down the center aisle and out the door to the Green. The sun was shining brilliantly, warmly. Spring had come at last.

We were as alone on the Green as we had been in the church. There were no cars, not even parked ones. She kissed me under the old maple. Her dress dropped away with an effortless gesture. We made love there in the grass, under the vernal sun. I let it happen. I had free will, and I let it happen because I firmly believed that it was *not* happening. In some other universe only a curtain's width away, Ariel and I were sitting in our pew, mindlessly registering the minister's droning cadences.

"I wish we could have had more time together," she said. "But I'm glad that at least we could do this. Bless you, Andy. Don't forget me."

An organ blast woke me up. I tried not to jump, but Shelly looked over at me and laughed. She knew I had been asleep.

And I had. That was all that had happened. I had been asleep. I had had a dream. I had not done anything of consequence. After the service I went into the bathroom to check my underwear, which was clean. I came out and had coffee. The kids had found a frisbee and wanted me to play on the Green, which I did. We all had lunch together at a restaurant, and then Shelly and I went home. I think I spread fertilizer on the lawn.

Sometime that afternoon, Ariel collapsed. George called us from the hospital. No one knew what the problem was, but they were guessing that it was some kind of stroke. She was

unconscious. The doctors couldn't say when she would wake up. We met George at the hospital, looked in on Ariel (who seemed to be just sleeping), and took the kids home with us. We bedded them down in our guest room that night, and I drove them to school in the morning. It was the least we could do.

Months went by. The doctors were calling it a coma now, but that was just a word. They didn't know what was wrong with her. Once a week I took an afternoon to sit by her bed and read to her, on the off-chance that she would hear. It was what she would have done for me. Though she would have been certain that I was listening, and I was anything but certain. About anything at all.

One day late in the fall, I didn't find Ariel in her usual room. I checked with several nurses, and I wound up walking down a long, white corridor. I couldn't understand why it was so long, because there were no other rooms on it. I began to wonder how long it would take, and if I should call Shelly to tell her that I would be late getting home. Finally I came to Ariel's room. It was very large and bright. Ariel was awake and alone. She was glad to see me.

Underneath the sheets was an enormous lump. "Is it a tumor?" I asked her. "Is that what the root problem is? Cancer?"

She laughed at me. I had forgotten how much I missed her laugh. There had been so little to laugh at this spring and summer and fall. But Ariel had never needed a reason to laugh, she just laughed whenever she wanted. It was her right.

"You silly thing," she said. "It's not a tumor, it's our child, our daughter. I've named her Miranda. I've been with her all these months, and I love her very much. You'll love her too, if you let yourself get to know her. Put your hand right there. I'm sure she'll kick for you."

I woke up from that dream in the middle of the night. I wanted to wake Shelly, but I didn't. It was just a dream, after all. But I went to the hospital that morning before my first lecture. Ariel was still unconscious, still in the same room. When I was certain that no one would see me, I lifted the covers and looked at her belly. She was frighteningly thin. There was no lump there, no tumor, no child.

December came, and we began planning Christmas with George and the kids. We would all take a trip to the hospital together, of course, but we wanted the kids to have as happy a day as we could give them. We wanted it not to be morbid. Shelly overspent our present budget by a wide margin, and wrapped them all as brightly as she could.

With only a few days to go, I found myself back in the long white corridor. Ariel was in a birthing room now. Her feet were in the stirrups and her stomach was enormous. "Where are the nurses?" I asked, and she told me I would have to make do without them. "That's absurd," I said, and I went out into the corridor to look for them.

"There's no time!" she yelled. Then she let loose with a half-grunt, half-scream that gave me the same message in a much more visceral form. I looked for call buttons I could push and strings I could pull, but there was nothing. I came back into the room, and for some indeterminate number of hours I held her hand, wiped her forehead, watched the top of the baby's head emerge, and told her how much I wanted them both to live.

I'm certain I did everything wrong.

From early on, there was far too much blood. I had never seen a birth before, but I was sure this couldn't be right. Miranda was soaked in blood when she fell into my hands. She was slick with it. I wrapped her in a towel and the towel turned red. "I can't stop your bleeding," I

told Ariel. “I have to find a doctor.”

“It’s too late for that,” she said. “Just take her. Don’t let her fade away.”

“I have to try to save you somehow.”

“Just take her!” Ariel yelled. “You’re the only one who can raise her. She’ll die without you. Just take her! Take her! Take her!”

I took her. I left Ariel lying in a pool of her own blood and I took our daughter out into the white corridor. I opened the only other door and found myself in the bedroom that I shared with Shelly. There was another door right next to my side of the bed, a door I had never seen before. I opened it and found a nursery there. I cleaned Miranda in the sink and put a new diaper on her. I wrapped her in a clean blanket and held her against my chest as I sat in the rocking chair next to the crib. I must have fallen asleep like that.

The telephone woke me up. It was George calling from the hospital. His voice was perfectly emotionless, which was always a bad sign with George. Ariel had suffered a massive abdominal hemorrhage. She was dead.

Christmas never happened that year. Ariel’s funeral was on the 27th. The church was full. The children looked drugged. I nodded off during the service just long enough to feed Miranda.

Shelly thought I was narcoleptic that winter, because I seemed to fall asleep every few hours. She made me see a neurologist, who had some theories but couldn’t turn up anything definite.

By spring George had found a new job in Texas. The children cried when they left and we all promised we’d stay in touch, but it was clear that George wanted to start a new life. I couldn’t deny him that.

In the end, Shelly missed the kids more than I did. One day in June I removed the brightly wrapped Christmas presents from her closet and took them to an orphanage. It was almost a year before she seemed like herself again. I tried to be there for her, but I’m afraid I didn’t do a very good job. I had way too much else to do.

I had Miranda.

*

Andrew was delusional.

Max had told my advisor as much right from the beginning, and Andrew had all but admitted it himself on several occasions, but I had found it convenient to forget. As long as we stuck to mathematics or the weather, he seemed so lucid, even wise. But he believed that for the last 19 years he had been raising a daughter in his dreams. He believed that she was there right this very minute, that he could visit her just by falling asleep.

What could I say to something like that?

It had taken hours for him to tell his story. The rain had stopped by the time he finished. Official sunset was still more than an hour away, but outside Andrew’s north-facing window things looked pretty dark. We sat silently in the shadows. Before long, I could tell from his breathing that he was asleep. He had earned it. We both had. I had kept him awake and talking far longer than Max had requested. I asked a nurse for help, and together we moved him out of his chair and onto the bed.

On the way home, I started wondering how I had recognized the resemblance between Ariel and the dazzling girl in my own dream. And I wondered how I had known to call her

Miranda. But I didn't worry about it much. I knew that memory is a tricky business, not at all the accurate permanent record that it seems to be. And memories of dreams are doubly tricky. I hadn't written down anything about the dream I had that first day by the pond. Maybe, as I listened to Andrew, I had been unconsciously rewriting my memory of the dream, making it correspond to what he was telling me. It sounded a little far-fetched, but compared to what?

One reason I didn't worry about it was that as I walked back to my room I had a new idea about my surface and my point. By the time I got home, all I wanted to do was pull out some of my notes from February and recalculate something that had seemed like a coincidence at the time. I fell asleep six hours later with papers scattered all over my bed.

I dreamed of Miranda, of course. She came up to my room and laughed at what a slob I was and what my refrigerator said about my eating habits. She told me I should take better care of myself. She said that life wasn't a sprint, that if it was any kind of race at all (which it probably wasn't) it was some kind of marathon. I barely stirred. She came over to the bed and swept some papers onto the floor. She sat next to me and stroked my hair. For some reason, that seemed like the most wonderful thing anyone had ever done for me. I felt like I could lie there forever.

When I woke up, my face was about six inches from the fan, which was blowing a single lock of hair back and forth across my forehead. Several of my papers had blown onto the floor.

Now, of course, it was no mystery that I dreamed about Miranda. Andrew had told me her story. I had seen the picture of the beautiful mother she supposedly resembled. I was a more-or-less healthy male in his late twenties who didn't have a girl friend and was working way too hard. What else would I dream about?

The next day was sunny and hot. I took Andrew back to the pond and we talked about mathematics. He seemed a little less focused than usual, but I had expected that. The previous day had been very emotional for him. I couldn't expect him to snap right back.

We both dozed in the sun for a while, and Miranda stopped by. She suggested skinny-dipping in the pond to cool off, but Andrew said that would be improper. She smiled, which made me wonder if she had suggested it just to gauge his reaction, or mine. She said that he didn't look well, and that she was worried about him.

When I woke up, Andrew was still dozing. I talked to him and he perked up a little, but still seemed groggy.

The next day he was a little better, and a little better again the day after. But he still seemed sluggish compared to a week before. I wondered if it was just my interpretation that was changing. Maybe I had exaggerated his sharpness before, and now, after seeing the full extent of his illness, I was over-reacting in the opposite direction.

As July turned into August, my life began to stabilize. More and more often, I worked by day, slept by night, and ate at least two decent meals. Miranda showed up in my dreams almost nightly. Some nights she read poetry to me. Some nights we argued about novels that I had read back in the days before mathematics took over my life. Sometimes we went for walks, and we usually ran into Andrew when we did. Occasionally we watched videos together, but when I woke up I could never remember what they had been about. When I was lazy, she pushed me. When I got discouraged, she expressed her faith in me.

By mid-August it was clear that Andrew was going downhill. I worried about him. (And

in my dreams I tried to divert Miranda, who worried about him too much.) He was still interested in my work, but more as a cheerleader now than as a participant. He remained convinced that I would find the surface and the point that I had been describing in my notes these many months. “You need to have faith in yourself and in what you can see with your own eyes,” he said. “Don’t wait for some authority to tell you what is true and what isn’t. Give yourself some credit. You deserve it.”

Paradoxically, as the real Andrew faded, my internalized Andrew picked up steam. There were days now where I stood back completely, and merely listened to the argument between my internalized advisor and my internalized Andrew. From time to time I would do a calculation to adjudicate some disagreement between them, but I did it with the impartiality of an umpire. It was as if the outcome of the argument was irrelevant to me; I only wanted to make sure it was true.

And then, one day in late August, I found it.

In the morning, I had made a revised list of the most important things I knew about the surface. It was bookkeeping, nothing more. I reduced ten messy pieces of paper to two neat ones. Then I went to see Andrew. The pond was exceptionally beautiful that day, and the ducks still had not learned. Andrew seemed to pay attention when I talked, but said very little in response. When I took him back to his room, he squeezed my hand and said, “Stick with it. You’ll get there.”

When I got back to my room, it occurred to me to wonder whether a set of equations of a certain form might define a surface that had most of the properties I had listed. It almost worked. I went for a walk, ate dinner at a cheap restaurant, and came back. I fiddled with the equations, half a dozen terms unexpectedly canceled out to zero, and there it was.

Once I had the surface, there weren’t many places for the point to hide. I had it before midnight. And I went to bed at a decent hour.

I tried to be modest and cautious about my claims, but Miranda saw through me immediately. She produced a bottle of champagne from somewhere. She cranked the stereo up and we danced. I worried that my neighbors would complain, but then I remembered that I was actually sound asleep and making no noise at all beyond a possible snore. We laughed about it. When it was time to go our separate ways, she let me kiss her good-bye. It seemed like the most romantic thing I had ever done.

In the morning, I started writing down a proof that someone other than myself might have a chance to follow. As I cleaned up my notation, I noticed that three of my choices had been arbitrary, and so I had a three-parameter family of examples rather than just one. For the first time in months, I thought about my rivals in Paris. The last I had heard, they had still been trying to prove the conjecture rather than disprove it, so I doubted they would beat me to the punch. But I began drafting an email to send them, so that there would be no doubt how early I had made my discovery.

That afternoon I had a great deal of trouble rousing Andrew. He didn’t want to get into his chair at all, and I decided not to try to force him. I sat on the side of his bed and held his hand. When he seemed alert, I told him that I had won, and that my advisor’s conjecture was false.

He smiled and nodded slowly. “The mind's eye can be very acute,” he said. “Never let them tell you that the things you see don’t exist.”

Before I left that evening, I got the head nurse to give me Max's number in Budapest. I waited until it was a decent hour of the morning in Hungary, and then I talked him into coming home a few days early. "If you wait until Labor Day, he might not be conscious any more," I said.

Max replied that he had been afraid this would happen, and he thanked me for notifying him, and for keeping Andrew going as long as I had.

"One thing I was curious about," I said just before hanging up. "Did you ever meet his daughter Miranda?"

There was a long silence on the line. "Miranda is part of Andrew's illness," Max said. "He never had a daughter. Miranda doesn't exist."

"That's what I had heard," I said. "But I wanted to double-check."

Things moved rather quickly after that. Max returned the next day, in time to say his good-byes. My advisor returned the day after. It took a full week to convince him that I was correct. Three times he thought he had overthrown my examples, but each time I was able to point out his mistake. The Parisians were easier. They sent me congratulations after two days.

My doctorate was now assured, but the academic hiring cycle wouldn't start in earnest until the spring. So I could, if I wanted, kick back and do little more than teach my calculus class. Already by the end of September my examples were known as "Murphy surfaces" on the internet discussion lists. My advisor was wondering how to salvage his system of conjectures, and I had a few ideas that both of us thought seemed promising. Keith was treating me more like an equal now. I had to admit that it felt good.

As I had predicted, Andrew became incoherent over the Labor Day weekend. I was off the payroll by now, but I went to his room once a day anyway, just to hold his hand and talk to him the way that he had talked to Ariel. But now we only conversed when he showed up with Miranda in my dreams. By the equinox he was completely comatose, and around the end of October he died. He had a huge funeral, attended by his colleagues from chemistry departments all over the world. There was some talk of starting an annual conference in his honor. I looked for Ariel's other children, but I didn't see them. Perhaps they didn't hear. Or maybe I just missed them in the crowd.

Sometime in mid-November, I got a call from a lawyer. Back in early July Andrew had rewritten his will to name me as his sole heir. I tried to guess what he must have been thinking then, but I couldn't. In theory someone could have contested the will, claiming either that Andrew was already of unsound mind at that point, or that I had exercised undue influence over him. But the truth was that no one else had a claim on Andrew's money. Max, I think, was actually relieved to hear that he was not the heir.

I was somewhat surprised to discover just how big an estate Andrew had left me. It wasn't captain-of-industry style money, but he had owned a few lucrative patents in addition to his academic retirement fund. It would be enough to keep me in pizzas for the foreseeable future. Probably I would never again have to take an odd job just to put a roof over my head.

Along with the money came an envelope. It contained a simple, handwritten note: "Take care of my girl. Don't let her fade away." That night I noticed a new door as I walked to my room. Miranda now had a room down the hall from me.

As I write this, it has just turned February. Everything is resolved now, except for Miranda. Over the last few months I have come to know her very well. I know her voice, her

laugh, her smile, her walk, her interests, her tastes.

The only thing I don't know is whether or not she exists.

Andrew thought she did, but Max was quite certain that she did not. Science doesn't seem to leave much space for her. She shows up in my dreams night after night, but how much does that really mean?

Like Gauss and Lewis Carroll, though, I have come to understand something: Not everything that seems ridiculous is actually impossible. Absurdity, I think, is a very poor reason for disbelieving in things, especially (as Andrew told me again and again) if you keep seeing them.

And so I am proceeding as I think any good mathematician would. I am learning everything I can about Miranda, and working out in my own mind what her existence would imply. As my vision of Miranda and her world gets clearer and clearer, someday I may come to see precisely what is wrong with that world, and why it can't exist.

Unless, of course, it does.